Eur. Phys. J. C 27, 145–152 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2002-01088-y

THE EUROPEAN
PHYSICAL JOURNAL C

On the relevance of matrix coordinates for the inside of baryons

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Received: 15 August 2002 /
Published online: 15 January 2003 –

Abstract. It is argued that one natural choice for the coordinates of the constituents of a baryonic state in a SU(N) gauge theory is the choice of $N \times N$ hermitian matrices. It is discussed that the relevance of matrix coordinates is supported at least by the restricted form of the color symmetry. Based on the previous investigations in this direction, the consequences of this idea are reviewed. The model has been considered in which it originates in the D0-branes of the string theory.

One of the main themes in quantum mechanics is to found our physical theories exclusively upon relationships between quantities which in principle are observable [1]. At present, it is commonly believed that a hadron has quarks as part of its ingredients, though they cannot be detected directly. From the pure theoretical point of view, one quark on its own is like the other particles, and has some observable quantities, such as position, momentum, spin or charge. In practice, seemingly we are always faced with the properties of quarks being hidden inside hadrons. Although it does not seem natural to assume that quarks do not carry any of the usual degrees of freedom or their degrees of freedom can be completely ignored, it may be a desirable framework – if it is possible – to assume that the degrees of freedom can become "unreachable" due to some kind of symmetry. In other words, due to a symmetry it would not be expected that, for example, the position of an individual quark can be measured. Or even the question about "the position of an individual quark with a specific color" becomes meaningless.

In [2–5], a model was considered which shares the feature we mentioned above. This model originates in the D0-branes [6,7] of string theory, for which it is known that their degrees of freedom are captured by matrices, rather than numbers [8]. The model one was concerned with in [2–5] has shown its ability to reproduce or cover some features and expectations in hadron physics. Some of these features and expectations are phenomenological inter-quark potentials, the behavior of total scattering amplitudes, a rich pology of the scattering amplitude, behavior in the large-N limit, and the whiteness of baryons with respect to the SU(N) sector of the external fields.

As mentioned, the internal dynamics of the D0-brane bound state is described by a matrix model of coordinates, and if the matrix coordinates of D0-branes have something to do with hadron physics, it is very logical to ask: "Is it possible to extract or derive these matrices from some first principles of quantum chromodynamics (QCD)?" In fact the answer to this question has been the motivation for the present work, and as we will see, it turns out that the appearance of matrix coordinates in the theory of quarks is as natural as the appearance in the theory of D0-branes. Before presenting the formal derivation, let us present the heuristic argument. Recalling the procedure of reasoning in D0-brane theory, we note that the matrix coordinates are the result of some states, to be specific some open string states, which are equipped with two more labels than the usual ones, the so-called Chan–Paton labels [8, 7,9]. In the open string picture these labels are attached to the ends of the string. On the other side, D0-branes are defined as point-like objects at whom the open strings end. By this picture, each D0-brane is accompanied with some more degrees of freedom than the usual ones of an ordinary particle. In other words, each D0-brane has some more degrees of freedom which express to which other D0-branes (and in what places) it is connected, i.e., has made a bound state with others. Eventually it appears that in a bound state of N D0-branes, the number of relevant degrees of freedom in each direction of space, rather than N, is N^2; this may be represented by a matrix belonging to the U(N) algebra. Now as we shall recognize in a moment, this reasoning is applicable for the case of quarks, in which the states have the "color" additional degrees of freedom.

In the constituent quark picture of a SU(N) gauge theory, a baryon is made by N quarks in different colors; and besides, to bring the baryon state to the form of a singlet in color space, anti-symmetrization in the color labels is understood. Let us for the moment forget about the rotation in the color space, and assume that the baryon is just made up of N quarks in different colors, represented by the states and wavefunctions $|\psi_a(t)\rangle$ and $\langle \mathbf{x}_a|\psi_a(t)\rangle = \psi_a(\mathbf{x}_a, t)$, $a = 1, \cdots, N$, respectively; in this work we also do not care about the fermionic or bosonic

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nature of the quarks. Though the wavefunctions depend on different arguments $\mathbf{x}_1, \cdots, \mathbf{x}_N$, while keeping the right form of each function ψ_a, we may represent them by one argument $\mathbf{x}$: $\psi_a(\mathbf{x}, t)$. By this we can define the $N \times N$ matrix $\mathbf{X}$ via its elements $\mathbf{X}_{ab}(t) \equiv \int \mathrm{d}\mathbf{x} \psi_b^*(\mathbf{x}, t)\mathbf{x}\psi_a(\mathbf{x}, t)$. Here we assume that the states are normalized properly, to yield the length dimension for the elements of $\mathbf{X}$, accompanied with the value 1 for the total probability. It is easily seen that $\mathbf{X}$ is a $N \times N$ hermitian matrix, and its elements are characterized by the color labels $a, b = 1, \cdots, N$.

Let us take the case for which we have well-separated quarks, which may be represented by the wavefunctions $\psi_a(\mathbf{x}, t) \simeq \delta(\mathbf{x} - \mathbf{x}_a)$ with $|\mathbf{x}_a - \mathbf{x}_b| \gg \ell$ $(a \neq b)$, for some characteristic length ℓ. For this case, the matrix $\mathbf{X}(t)$ is almost, or even in this case exactly, diagonal. Suppose we take the length scale ℓ to be the order of the baryon size. From our experience, we know that the situation we have considered above is never seen in practice! The most expected situation is that, due to confinement, the N quarks have considerable overlap between their wavefunctions and form a baryon. Correspondingly, we have learnt to deal always with permanently “connected” quarks, for which the matrix $\mathbf{X}(t)$ appears always in non-diagonal form, and this may cause one to believe in the essence of more degrees of freedom as representatives and also in the description of permanent connectivity of the quarks. We note that in fact a huge amount of information about the inside of a baryon is encoded in the wavefunctions of its constituents, or equivalently in the matrix coordinate $\mathbf{X}$ and its generalizations to higher moments $\mathbf{X}_{ab}^{(n)}(t) \equiv \int \mathrm{d}\mathbf{x}\psi_b^*(\mathbf{x}, t)\mathbf{x}^n\psi_a(\mathbf{x}, t)$, $n = 0, 2, 3, 4, \cdots$ The above simple observation may suggest that the matrix coordinate $\mathbf{X}$ and its generalizations to higher moments can lead to the criteria for the identification of the confined phase of the theory. Besides, the matrix coordinate $\mathbf{X}$ and its higher moments may be taken as a set of very powerful tools for the characterization and the study of the observable states in a confined theory. Therefore, it will be very tempting to see by considering the matrix $\mathbf{X}(t)$ as the dynamical variable relevant for the inside of a baryon, what kind of information and conceptual insights come out.

Before proceeding, it is useful to mention that the matrix coordinate can also be constructed from the original quark field in the Lagrangian. We take a SU(N) gauge theory, consisting of one kind of flavor in the fundamental matter representation. We treat this example as quantum mechanics, rather than a field theory. The states of matter in this quantum mechanical problem are represented by

$$|\Psi(t)\rangle = \begin{pmatrix} |\psi_1(t)\rangle \\ |\psi_2(t)\rangle \\ \vdots \\ |\psi_N(t)\rangle \end{pmatrix}. \qquad (1)$$

So we have the expansion $|\Psi(t)\rangle = \int \mathrm{d}\mathbf{x} \sum_{a=1}^N \psi_a(\mathbf{x}, t)|\mathbf{x}\rangle \otimes |a\rangle$, in which the index a is labelling the isospin, and $\psi_a(\mathbf{x}, t) \equiv \langle \mathbf{x}|\psi_a(t)\rangle$. We define the density matrix operator $\hat{\rho}(t)$ by

$$\hat{\rho}(t) \equiv |\Psi(t)\rangle\langle\Psi(t)|, \qquad (2)$$

which is an $N \times N$ matrix with the general element $\hat{\rho}_{ab}(t) = |\psi_a(t)\rangle\langle\psi_b(t)|$. By the density operator $\hat{\rho}_{ab}(t)$, we can evaluate a particular expectation value for the position operator simply by

$$\mathbf{X}(t) \equiv \mathrm{tr}_{\mathbf{x}}(\hat{\mathbf{x}}\hat{\rho}(t)) = \qquad (3)$$
$$\begin{pmatrix} \langle\psi_1(t)|\hat{\mathbf{x}}|\psi_1(t)\rangle & \langle\psi_2(t)|\hat{\mathbf{x}}|\psi_1(t)\rangle & \ldots & \langle\psi_N(t)|\hat{\mathbf{x}}|\psi_1(t)\rangle \\ \langle\psi_1(t)|\hat{\mathbf{x}}|\psi_2(t)\rangle & \ldots & \ldots & \ldots \\ \vdots & \ddots & \ddots & \vdots \\ \langle\psi_1(t)|\hat{\mathbf{x}}|\psi_N(t)\rangle & \ldots & \ldots & \langle\psi_N(t)|\hat{\mathbf{x}}|\psi_N(t)\rangle \end{pmatrix},$$

in which $\hat{\mathbf{x}}$ is the usual position operator, and $\mathrm{tr}_{\mathbf{x}}$ means the integration on the volume of space, yielding

$$\langle\psi_a(t)|\hat{\mathbf{x}}|\psi_b(t)\rangle = \int \mathrm{d}\mathbf{x}\psi_a^*(\mathbf{x}, t)\mathbf{x}\psi_b(\mathbf{x}, t).$$

The general element is defined by

$$\mathbf{X}_{ab}(t) = \langle\psi_b(t)|\hat{\mathbf{x}}|\psi_a(t)\rangle = \mathbf{X}_{ba}^*(t),$$

and so the matrix coordinate $\mathbf{X}(t)$ is a $N \times N$ hermitian matrix with the usual expansion in color (isospin) space $\mathbf{X}(t) = \sum_{a,b=1}^N \mathbf{X}_{ab}(t)|a\rangle\langle b|$. Again as is easily recognized, the elements of the matrix coordinate $\mathbf{X}$ are characterized by the color labels $a, b = 1, \cdots, N$.

As usual, it is natural to assume that the expectation values satisfy some classical equations of motion. Also, we expect that via the quantization of the resultant classical theory, we end up with the original quantum theory. In the general case, one expects that the classical equations can be derived from the quantized theory, in particular by the equations of motion for the states or wavefunctions. Since in the problem at hand the quantum theory, especially in the non-perturbative regime, is too hard to solve, one may try to formulate the classical theory on some general grounds. In our specific case we are naturally faced with a matrix model. So the general classical action for the coordinates $\mathbf{X}(t)$ may be taken to be

$$S[\mathbf{X}] = \int \mathrm{d}t \mathrm{Tr}\left(\frac{1}{2}m\dot{\mathbf{X}} \cdot \dot{\mathbf{X}} - \mathcal{V}(\mathbf{X}, \dot{\mathbf{X}}, \mathbf{X}_{ab}, \dot{\mathbf{X}}_{ab})\right), \qquad (4)$$

where Tr acts on the matrix structure, and “$\mathcal{V}(\cdots)$” is for the possible potential term, depending on matrix coordinate or velocity, or probably some of their individual elements $\mathbf{X}_{ab}$ and $\dot{\mathbf{X}}_{ab}$. For the well-separated quarks, as was mentioned before, the coordinate matrix $\mathbf{X}(t)$ is almost, or in this case even exactly, diagonal and the action (4) becomes

$$S[\mathbf{X}] \simeq S[\mathbf{x}_1, \cdots, \mathbf{x}_N] = \int \mathrm{d}t \sum_{a=1}^N \left(\frac{1}{2}m\dot{\mathbf{x}}_a \cdot \dot{\mathbf{x}}_a - \cdots\right), \qquad (5)$$

in which $\mathbf{x}_a = \mathbf{X}_{aa}$. The kinetic term of the action (5) may be interpreted as the kinetic term of the N quarks. This shows that our new tools, the matrix coordinates, consist of the information we usually realize, in particular the positions and velocities of the individual quarks.

The issue of gauge symmetry of the original quantum mechanical problem should be considered. The theory we start with is invariant under the transformations

$$\begin{aligned}|\Psi(t)\rangle &\to |\Psi'(t)\rangle = \hat{V}(\hat{\mathbf{x}},t)|\Psi(t)\rangle,\\ \langle\Psi(t)| &\to \langle\Psi'(t)| = \langle\Psi(t)|\hat{V}^\dagger(\hat{\mathbf{x}},t),\\ \hat{\mathcal{O}}(\hat{\mathbf{x}},\hat{\mathbf{p}},t,\partial_t) &\to \hat{\mathcal{O}}'(\hat{\mathbf{x}},\hat{\mathbf{p}},t,\partial_t)\\ &= \hat{V}(\hat{\mathbf{x}},t)\hat{\mathcal{O}}(\hat{\mathbf{x}},\hat{\mathbf{p}},t,\partial_t)\hat{V}^\dagger(\hat{\mathbf{x}},t),\end{aligned} \quad (6)$$

for the Hamiltonian of the form $\mathcal{H} = \langle\Psi(t)|\hat{\mathcal{O}}(\hat{\mathbf{x}},\hat{\mathbf{p}},t,\partial_t)|\Psi(t)\rangle$, and $\hat{V}(\hat{\mathbf{x}},t)$ is an $N\times N$ unitary operator, i.e., $\hat{V}\hat{V}^\dagger = \hat{V}^\dagger\hat{V} = \mathbf{1}_N$. Due to integration on the space $\int \mathrm{d}\mathbf{x}$, it might not be expected that in a simple way the whole large symmetry above can be recovered in the theory for matrix coordinates. Instead we assume that the position dependence of the $\hat{V}$ matrix is in the form of $\hat{V}(\hat{\mathbf{x}},t) = \tilde{U}(\hat{\mathbf{x}})U(t)$, where $U(t)$ is an $N\times N$ unitary matrix, and $\tilde{U}(\hat{\mathbf{x}})$ is a phase depending on the position operator $\hat{\mathbf{x}}$, i.e., $\tilde{U}\tilde{U}^* = 1$. By this kind of transformations we are treating the position dependence of matrix $\hat{V}$ as a U(1) group, rather than a non-Abelian one. Later we will try to present some kind of justification for the restriction on the transformations. It can be seen that the matrix coordinate transforms as $\mathbf{X}(t) \to \mathbf{X}'(t) = U(t)\mathbf{X}(t)U^\dagger(t)$. So our matrix theory, at least, should be invariant under this kind of transformations[1], and as usual this can be done by introducing a covariant derivative. So the action (4) can be rewritten as

$$S[a_t,\mathbf{X}] = \int \mathrm{d}t\mathrm{Tr}\left(\frac{1}{2}mD_t\mathbf{X}\cdot D_t\mathbf{X} - \mathcal{V}(\mathbf{X}, D_t\mathbf{X})\right), \quad (7)$$

in which $D_t\mathbf{X} = \dot{\mathbf{X}} + \mathrm{i}[a_t,\mathbf{X}]$, with $a_t(t)$ the one dimensional $N\times N$ hermitian gauge field. We see that the action is now invariant under the transformations

$$\begin{aligned}\mathbf{X} &\to \mathbf{X}' = U\mathbf{X}U^\dagger,\\ a_t &\to a_t' = Ua_tU^\dagger - \mathrm{i}U\partial_tU^\dagger,\end{aligned} \quad (8)$$

with $U \equiv U(t)$ an arbitrary $N\times N$ unitary matrix; in fact under these transformations one obtains

$$\begin{aligned}D_t\mathbf{X} &\to D_t'\mathbf{X}' = U(D_t\mathbf{X})U^\dagger,\\ D_tD_t\mathbf{X} &\to D_t'D_t'\mathbf{X}' = U(D_tD_t\mathbf{X})U^\dagger.\end{aligned} \quad (9)$$

One may go a little more in details on the potential term. First, we assume that the potential is linear in the velocity $D_t\mathbf{X}$, appearing in the potential as $D_t\mathbf{X}\cdot\mathbf{A}(\mathbf{X},t)$. Second, since here we have matrices as coordinates, we can decompose the velocity independent term to completely symmetric and non-symmetric parts in components of $\mathbf{X} = (X^1, X^2, \cdots, X^d)$. We note that each component X^i is a matrix. The non-symmetric part can be expanded as

$$\begin{aligned}&\mathcal{V}_{\frac{\text{veloc. indepen.}}{\text{non-symm.}}}(\mathbf{X})\\ &= \underbrace{X^i + [X^i,X^j] + X^i[X^j,X^k]}\\ &- \frac{m}{4l^4}[X^i,X^j][X_i,X_j] + O(X^6),\end{aligned} \quad (10)$$

in which the terms "$\underbrace{\cdots}$" consist of free space indices or traceless parts. So the first surviving term is "$-m[X^i, X^j]^2/4l^4$", with l a proper length scale. Finally, we require that the vector potential $\mathbf{A}(\mathbf{X},t)$ is also symmetric in the components X^i. Putting these all into the potential term, we end up with the action

$$\begin{aligned}S[a_t,\mathbf{X}] = \int \mathrm{d}t\mathrm{Tr}\bigg(&\frac{1}{2}mD_t\mathbf{X}\cdot D_t\mathbf{X} + qD_t\mathbf{X}\cdot\mathbf{A}(\mathbf{X},t)\\ &- qA_0(\mathbf{X},t) + \frac{m}{4l^4}[X^i,X^j]^2\bigg),\end{aligned} \quad (11)$$

in which $A_0(\mathbf{X},t)$ is the symmetric part of the velocity independent term of the potential, and q plays the role of the charge. We note that the fields $(A_0(\mathbf{X},t), \mathbf{A}(\mathbf{X},t))$ appear as $N\times N$ hermitian matrices due to their functional dependence on the matrix coordinate $\mathbf{X}$. It is interesting to study the gauge symmetry of this action. One can check easily that the action (11) is invariant under the symmetry transformations [10–12]

$$\begin{aligned}\mathbf{X} &\to \mathbf{X}' = U\mathbf{X}U^\dagger, \quad (12)\\ a_t(t) &\to a_t'(t) = Ua_t(t)U^\dagger - \mathrm{i}U\frac{\mathrm{d}}{\mathrm{d}t}U^\dagger,\\ A_i(\mathbf{X},t) &\to A_i'(\mathbf{X}',t) = UA_i(\mathbf{X},t)U^\dagger + \mathrm{i}U\delta_iU^\dagger,\\ A_0(\mathbf{X},t) &\to A_0'(\mathbf{X}',t) = UA_0(\mathbf{X},t)U^\dagger - \mathrm{i}U\partial_tU^\dagger,\end{aligned}$$

where $U \equiv U(\mathbf{X},t) = \exp(\mathrm{i}\Lambda)$ is arbitrary up to the condition that $\Lambda(\mathbf{X},t)$ is hermitian and totally symmetrized in the X^i. Above, δ_i is the functional derivative $\delta/\delta X^i$, and we note that though $U(\mathbf{X},t)$ depends on $\mathbf{X}(t)$, due to the total derivative $\mathrm{d}/\mathrm{d}t$, $a_t'(t)$ still only depends on time. We recall that, in approving the invariance of the action, the symmetrization prescription on the matrix coordinates plays an essential role [10,11]. It is by this symmetry transformation that we expect that no distinguished role should be identified to the (diagonal or off-diagonal) elements of the matrix coordinate. In other words, since none of the matrix elements are gauge invariant quantities, they are not expected to appear as an observable final state.

The above transformations on the gauge potentials are similar to those of non-Abelian gauge theories, and we mention that this is just a consequence of the enhancement of the degrees of freedom from numbers ($\mathbf{x}$) to matrices ($\mathbf{X}$). In other words, we are faced with a situation in which "the rotation of fields" is generated by "the rotation of coordinates" [11]. In addition, the case we see here may be considered as a finite-N version of the relation between gauge symmetry transformations and transformations of matrix coordinates [13]. Despite the non-Abelian behavior of the gauge transformations, we should note that the symmetry is still not equivalent to a non-Abelian one. To see this, we should recall that the symmetry transformations

[1] The invariance under the global transformations by $\hat{V}(\hat{\mathbf{x}},t) = V_0$, with V_0 a constant $N\times N$ unitary matrix, requires that the action should not consist of the individual elements of $\mathbf{X}$, as we assumed in (4), in the first step

of, for example, a U(N) gauge theory is generated by N^2 functions of space-time, say $\Lambda_\alpha(\mathbf{x},t)$ ($\alpha = 0,\cdots,N^2-1$), in the group element $\exp(\mathrm{i}\Lambda_\alpha T^\alpha)$, where the T^α are U(N) generators. Now although $U(\mathbf{X},t)=\exp(\mathrm{i}\Lambda(\mathbf{X},t))$ in (12) is a unitary matrix due to its dependence on the matrix coordinate, it is constructed by just one function $\Lambda(\mathbf{x},t)$, after replacing the coordinates by matrices i.e. $\mathbf{x}\to\mathbf{X}$, under the condition of symmetrization. After all, it is quite natural to interpret the fields $(A_0,\mathbf{A})$ as the external gauge fields that the constituents, whose degrees of freedom are included in the matrix coordinate, interact with.

The action (11) is known to be the action of N D0-branes of string theory, in the background of the (RR) gauge field $(A_0(\mathbf{x},t),\mathbf{A}(\mathbf{x},t))$, for $\mathbf{x}$ as the ordinary coordinates [14]. As mentioned before, from the string theory point of view, D0-branes are point particles to which ends of strings are attached [6]. In a bound state of N D0-branes, they are connected to each other by strings stretched between them, and it can be shown that, by counting the degrees of freedom for the oriented strings, the correct dynamical variables describing the positions of D0-branes are $N\times N$ hermitian matrices [8]. By comparison, we find that m is the mass of D0-branes and l is the order of the string length. In [2–5] the possibility for the identification of the dynamics of D0-branes and quarks are investigated. Here we recall some of the aspects mentioned in these papers. First of all, we see that by the gauge transformation (12), the elements of the position matrix mix with each other, and so the interpretation of the positions for the D0-branes remains obscure. Nevertheless, we note that the concept of center-of-mass (c.m.), here represented by the trace of the matrix coordinate, is meaningful. So the ambiguity of the positions only remains for the degrees of freedom counting the relative positions of the D0-branes and the strings stretched between them. The equations of motion for the X^i and a_t by the action (11), ignoring the commutator potential $[X_i,X_j]^2$, is found to be [10–12]

$$mD_tD_tX_i = q(E_i(\mathbf{X},t) + \underbrace{D_tX^jB_{ji}(\mathbf{X},t)}), \quad (13)$$

$$m[X_i, D_tX^i] = q[A_i(\mathbf{X},t), X^i], \quad (14)$$

with the following definitions:

$$E_i(\mathbf{X},t) \equiv -\delta_i A_0(\mathbf{X},t) - \partial_t A_i(\mathbf{X},t), \quad (15)$$

$$B_{ji}(\mathbf{X},t) \equiv -\delta_j A_i(\mathbf{X},t) + \delta_i A_j(\mathbf{X},t). \quad (16)$$

In (13), the symbol $\underbrace{D_tX^jB_{ji}(\mathbf{X},t)}$ denotes the average over all of the positions of D_tX^j over the X of $B_{ji}(\mathbf{X},t)$. The above equations for the X are like the Lorentz equations of motion, with the exceptions that the two sides are $N\times N$ matrices, and the time derivative ∂_t is replaced by its covariant counterpart D_t.

The behavior of (13) and (14) under the gauge transformation (12) can be checked. Since the action is invariant under (12), it is expected that the equations of motion change covariantly. The left-hand side of (13) changes to $U^\dagger D_tD_tXU$ by (9), and therefore we should find the same change for the right-hand side. One can check that in fact this is the case [10–12], and consequently one finds that (16) leads to

$$E_i(\mathbf{X},t)\to E'_i(\mathbf{X}',t) = UE_i(\mathbf{X},t)U^\dagger,$$
$$B_{ji}(\mathbf{X},t)\to B'_{ji}(\mathbf{X}',t) = UB_{ji}(\mathbf{X},t)U^\dagger. \quad (17)$$

This result is consistent with the fact that E_i and B_{ji} are functionals of the X. We thus see that, in spite of the absence of the usual commutator term $\mathrm{i}[A_\mu,A_\nu]$ of non-Abelian gauge theories, in our case the field strengths transform like non-Abelian ones. We recall that these are all consequences of the matrix coordinates of the D0-branes. Finally by a similar reason, vanishing of the second term of (11), both sides of (14) transform identically.

An equation of motion similar to (13) is considered in [5,4] as a part of similarities between the dynamics of D0-branes and bound states of quarks–QCD strings in a baryonic state [5,4,2]. The point is that the dynamics of the bound state c.m. is not affected directly by the non-Abelian sector of the background, i.e., the c.m. is "white" with respect to the SU(N) sector of the matrices. The c.m. coordinates and momenta are defined by

$$\mathbf{x}_{\text{c.m.}} \equiv \frac{1}{N}\mathrm{Tr}\mathbf{X}, \quad \mathbf{p}_{\text{c.m.}} \equiv \mathrm{Tr}\mathbf{P}, \quad (18)$$

where we are using the convention $\mathrm{Tr}\mathbf{1}_N = N$. To specify the net charge of a bound state (which is an extended object) its dynamics should be studied in zero magnetic and uniform electric fields, i.e., $B_{ji}=0$ and $E_i(\mathbf{X},t)=E_{0i}$[2]. Since the fields are uniform, they do not involve the X^i matrices, and contain just the U(1) part. In other words, under gauge transformations E_{0i} and $B_{ji}=0$ transform to $E'_i(\mathbf{X},t) = U(\mathbf{X},t)E_{0i}U^\dagger(\mathbf{X},t) = E_{0i}$ and $B'_{ji}=0$. Thus the action (11) yields the following equation of motion:

$$(Nm)\ddot{\mathbf{x}}_{\text{c.m.}} = Nq\mathbf{E}_{0(1)}, \quad (19)$$

in which the subscript (1) emphasizes the U(1) electric field. So the c.m. interacts directly only with the U(1) of U(N). From the string theory point of view, this observation is based on the simple fact that the SU(N) structure of the D0-branes arises just from the internal degrees of freedom inside the bound state. In other words, the matrix behavior of the coordinates, and the resulting non-commutativity, is just restricted to the relative positions of the D0-branes. In this picture, we may call this situation "confined non-commutativity" [12,11,5,4]. This behavior of D0-brane bound states is the same as that of baryons. It means that each D0-brane feels the net effect of other D0-branes as the white-complement of its color. In other words, the field flux extracted from one D0-brane to the other ones are the same as the flux between a color and an anti-color, Fig. 1. This shape for the electric flux is in agreement with the result of the field theory correlator

[2] In a non-Abelian gauge theory a uniform electric field can be defined up to a gauge transformation, which is sufficient for the identification of white (singlet) states

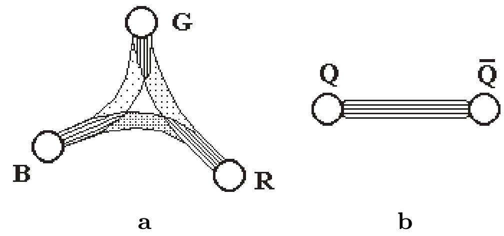

Fig. 1a,b. The net electric flux extracted from each quark is equivalent in a baryon **a** and a meson **b**. The D0-brane–quark correspondence suggests the string-like shape for the flux inside a baryon **a**

method [15]. It was pointed out that the gauge symmetry associated to the gauge field $(A_0(\mathbf{X},t),\mathbf{A}(\mathbf{X},t))$, though looking similar to the non-Abelian gauge theories, is intrinsically U(1). Based on the observation we have made here about the whiteness of the bound state, we may argue that in this phase all of the observable states should have an equivalent amount of U(N) sectors, the symmetry appears to be restricted, and equivalently for U(1). In fact this is the case that we expect to see when we are faced with matrix coordinates as the relevant degrees of freedom.

It is desirable to assign a net charge different from Nq to the c.m. This can be done simply by modifying the action (11):

$$S'[a_t,\mathbf{X}] = S[a_t,\mathbf{X}] + \int \mathrm{d}t (Nq'\dot{\mathbf{x}}_{\mathrm{c.m.}}\cdot\mathbf{A}(\mathbf{x}_{\mathrm{c.m.}},t) - Nq'A_0(\mathbf{x}_{\mathrm{c.m.}},t)), \tag{20}$$

in which $S[a_t,\mathbf{X}]$ is the action (11). With this action the charge of the c.m. is equal to $N(q+q')$, rather than Nq.

Now, let us ignore for the moment the external gauge field $(A_0,\mathbf{A})$. The equations of motion can be solved by diagonal configurations, such as

$$\mathbf{X}(t) = \mathrm{diag.}(\mathbf{x}_1(t),\cdots,\mathbf{x}_N(t)),$$
$$a_t(t) = \mathrm{diag.}(a_{t1}(t),\cdots,a_{tN}(t)), \tag{21}$$

with $\mathbf{x}_a = \mathbf{x}_{a0} + \mathbf{v}_a t$, $a = 1,\cdots,N$. In this configuration, we restrict the U(N) generators to the N dimensional Cartan subalgebra. This configuration describes the "classical" free motion of N D0-branes, neglecting the effects of the strings (and the symmetry supported by them). Of course the situation is different when we consider quantum effects, and consequently it will be realized that the dynamics of the off-diagonal elements affect the dynamics of the D0-branes significantly. Concerning the effect of the strings, one may try to extract the effective theory for D0-branes, i.e., for the diagonal configurations. In particular, it will be found that the commutator potential is responsible for the formation of the bound state, and by a simple dimensional analysis we understand that the size of the bound state, ℓ, is $\sim m^{-1/3}l^{2/3}$. As in [2] (see also [4,5]), let us take the example of static D0-branes. For this configuration one can easily calculate the one-loop effective potential between the quarks, getting [4,5,2]

$$V_{\mathrm{one-loop}} \sim 4\pi\frac{d-1}{2}\sum_{a>b=1}^{N}\frac{|\mathbf{x}_a-\mathbf{x}_b|}{l^2}. \tag{22}$$

This result shows the linear potential between each pair of D0-branes. Previously we mentioned, by qualitative considerations, what should be the shape of the electric flux (Fig. 1). Now, by the interpretation of (22) as the effective potential of a constituent quark model, we are enabled to get to know something more about the bound state and more quantitative details. One can trace support for the linear behavior of the potential in the literature, namely results by lattice calculations [16,17], and things we expect from the spin–mass Regge trajectories. In [18] by taking the linear potential between the quarks of a baryonic state in the transverse direction of the light-cone frame, the structure functions obtained are in good agreement with the observed ones. Since the original theory is invariant under rotation among the color indices $1,\cdots,N$, we mention that only the states which are singlets under the (global) rotation among the indices can be accepted as the physical states of the effective theory for diagonal elements.

The formulation we presented above is in the non-relativistic limit. Though it is expected that this limit produces good results for heavy quarks, for light or massless quarks we should change our approach. One way can be starting by a covariant theory; treating time and space equivalently. In this way, although the terms responsible for kinetic energy and interaction with external gauge fields get reasonable forms (see [11,12]), the main problem will appear to be with potentials as $[X^\mu,X^\nu]^2$. Instead one may follow another approach to say something about the covariant theory. The worldline formulation we have here is that of the M(atrix) model conjecture [19, 20], accompanied with the interaction terms with external gauge fields. For the case of the dynamics of a massless charged particle with ordinary coordinates, we can see easily that the light-cone dynamics have a form similar to what we have in the action (11); see the appendix of [4]. To approach the covariant formulation, following the finite-N interpretation of [21], it is reasonable to interpret things in the DLCQ framework [3–5,12]. In this interpretation, the mass parameter m is the longitudinal momentum, and the spatial directions present the transverse coordinates in the light-cone frame. In addition, according to the specific form of the action (11) the rest mass of quarks is assumed to be zero (see [4,5]).

In [3,4,12] the problem of scattering of

(1) a D0-brane off another one, and
(2) a D0-brane bound state off an external gauge field probe, were considered.

As we mentioned above, both of these scattering processes can be interpreted in the light-cone frame. For the case of scattering of a D0-brane off another one, the expectations for the well-known Regge behavior are satisfied. As for the problem of an interaction between the D0-brane bound state and "photons" of the gauge field, interesting observations are expected for the regime in which the details of the bound state can be probed. Here we just present the general expected features; see [12] for more details. As argued before, the external field depends on the internal coordinates of the bound state under the symmetrization condition in the matrix coordinates. One way to cover the symmetrization is to use the so-called "non-Abelian Fourier expansion" [12]. For an arbitrary function $f(\mathbf{X}, t)$ the non-Abelian Fourier expansion will be found to be

$$f(\mathbf{X}, t) = \int \mathrm{d}\mathbf{k} \bar{f}(\mathbf{k}, t) \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{X}}, \tag{23}$$

in which $\bar{f}(\mathbf{k}, t)$ are the Fourier components of the function $f(\mathbf{x}, t)$ (i.e., a function by of ordinary coordinates) which is defined by the known expression

$$\bar{f}(\mathbf{k}, t) \equiv \frac{1}{(2\pi)^d} \int \mathrm{d}\mathbf{x} f(\mathbf{x}, t) \mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}}. \tag{24}$$

Since the momentum numbers k_i are ordinary numbers and so commute with each other, the symmetrization prescription is automatically recovered in the expansion of the momentum eigen-functions $\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{X}}$. Now, by using the symmetric expansion (23), we can imagine some general aspects of the interaction between the D0-brane bound states and RR photons. As we mentioned before the size of the bound state, for a finite number N of D0-branes is finite and is of the order of $\ell \sim m^{-1/3} l^{2/3}$.

Before proceeding, we should distinguish the dynamics of the c.m. from the internal degrees of freedom of the bound state. As mentioned before, the c.m. position and momentum of the bound state are represented by the U(1) sector of the $\mathrm{U}(N) = \mathrm{SU}(N) \times \mathrm{U}(1)$, and thus the information related to the c.m. can be gained simply by the Tr-operation. So the internal degrees of freedom of the bound state, which consist of the relative positions of N D0-branes together with the dynamics of strings stretched between the D0-branes, are described by the SU(N) sector of the matrix coordinates. It is easy to see that the commutator potential in the action has some flat directions, along which the eigenvalues can take arbitrary large values. But it is understood that, by considering the quantum effects and in the case that we expect the formation of the bound state, we should expect suppression of the large values of the internal degrees of freedom [22]. Consequently, it is expected that the SU(N) sector of the matrix coordinates take mean values like $\langle X^i_\alpha \rangle \sim \ell$ ($\alpha = 1, \cdots, N^2 - 1$, not $\alpha = 0$ as for the c.m.), with ℓ as the bound state size scale mentioned in above. We should mention that, though the c.m. is represented by the U(1) sector, its dynamics is affected by the interaction of the ingredients of the bound state with the SU(N) sector of external fields, similar to the situation we imagine in the case of the Van der Waals force.

The important question about the interaction of a bound state (as an extended object) with an external field, is of "the regime in which the substructure of bound state is probed". As we mentioned in the introduction, in our case the quanta of the RR fields are the representatives of the external field. The quanta are coming from a "source", and so as it makes things easier, we ignore its dynamics.

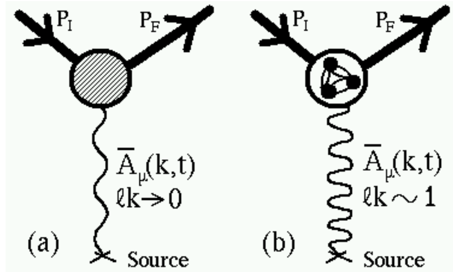

Fig. 2a,b. Substructure is not seen by the long wavelength modes **a**. Due to functional dependence on the matrix coordinates, the short wavelength modes can probe the inside of the bound state **b**. ℓ and $\bar{A}_\mu(k, t)$ represent the size of the bound state and the Fourier modes, respectively

The source is introduced into our problem by the gauge field $A_\mu(\mathbf{x}, t)$. These fields appear in the action by functional dependence on the matrix coordinates $\mathbf{X}$. In fact this is the key of how we can probe the substructure of the bound state. According to the non-Abelian Fourier expansion we mentioned above, we have

$$A_\mu(\mathbf{X}, t) = \int \mathrm{d}\mathbf{k} \bar{A}_\mu(\mathbf{k}, t) \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{X}}, \tag{25}$$

in which the $\bar{A}_\mu(\mathbf{k}, t)$ are the Fourier components of the fields $A_\mu(\mathbf{x}, t)$ (i.e., fields with ordinary coordinates). One can imagine the scattering processes which are designed to probe the inside of the bound state. As for every other scattering process, the two limits of the momentum modes corresponding to long and short wavelengths behave differently.

In the limit $\ell|\mathbf{k}| \to 0$ (long wavelength regime), the field A_μ is not involved via the $\mathbf{X}$ matrices mainly. This means that the fields appear to be nearly constant inside the bound state, and in a rough estimation we have

$$\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{X}} \sim \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{X}_{\mathrm{c.m.}}}. \tag{26}$$

So in this limit we expect that the substructure and consequently non-commutativity will not be seen; Fig. 2a. As a consequence, after interaction with a long wavelength mode, it is not expected that the bound state will jump to another energy level different from the first one. It should be noted that the c.m. dynamics can be affected as well in this case.

In the limit $\ell|\mathbf{k}|$ = finite (short wavelength regime), the fields depend on the coordinates $\mathbf{X}$ inside the bound state, and so the substructure responsible for non-commutativity should be probed; Fig. 2b. In fact, we know that the non-commutativity of the D0-brane coordinates is a consequence of the strings which are stretched between the D0-branes. In this case, it is completely expectable that the energy level of the incoming and outgoing bound states will be different, since the ingredients of the bound state substructure can absorb quanta of energy from the incident wave. In this case the c.m. dynamics can be affected in a novel way by the interaction of the substructure with the external fields (the Van der Waals effect). In the general case, one can gain more information about the substructure of a bound state by analyzing the "recoil" effect on the source. To do this, one should be able to include the dynamics of the source in the formulation. Considering the dynamics of the source in the terms of quantized field theory means that we consider the processes in which the source and the target exchange "one quantum of gauge field" with definite wavelength and frequency, though off-shell, as $A_\mu(\mathbf{x},t) \sim \epsilon_\mu \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}-\mathrm{i}\omega t}$.

Up to now, we have considered things for the theory with one kind of flavor. It is interesting to think about the case with more than one flavor. One suggestion can be as follows: assume that the flavor A with mass m_A is represented by the state $|\Psi_A(t)\rangle$. We may re-scale the states as $|\Psi_A\rangle \to |\tilde{\Psi}_A\rangle = (m_A)^{1/4}|\Psi_A\rangle$. For a baryon consisting of N heavy flavors we define the matrix coordinate by

$$\tilde{\mathbf{X}}(t) \equiv \begin{pmatrix} \langle\tilde{\psi}_1(t)|\hat{\mathbf{x}}|\tilde{\psi}_1(t)\rangle & \langle\tilde{\psi}_2(t)|\hat{\mathbf{x}}|\tilde{\psi}_1(t)\rangle & \dots & \langle\tilde{\psi}_N(t)|\hat{\mathbf{x}}|\tilde{\psi}_1(t)\rangle \\ \langle\tilde{\psi}_1(t)|\hat{\mathbf{x}}|\tilde{\psi}_2(t)\rangle & \dots & \dots & \dots \\ \vdots & \ddots & \ddots & \vdots \\ \langle\tilde{\psi}_1(t)|\hat{\mathbf{x}}|\tilde{\psi}_N(t)\rangle & \dots & \dots & \langle\tilde{\psi}_N(t)|\hat{\mathbf{x}}|\tilde{\psi}_N(t)\rangle \end{pmatrix}. \tag{27}$$

For this coordinate we take the action

$$S[\tilde{\mathbf{X}}] = \int \mathrm{d}t \mathrm{Tr}\left(\frac{1}{2}\dot{\tilde{\mathbf{X}}}\cdot\dot{\tilde{\mathbf{X}}} - \cdots\right). \tag{28}$$

Now, for the well-separated states, for which we have diagonal coordinates, the action in terms of the original coordinates (i.e., before re-scaling) becomes

$$S[\mathbf{x}_A] = \int \mathrm{d}t \sum_{A=1}^{N}\left(\frac{1}{2}m_A\dot{\mathbf{x}}_A\cdot\dot{\mathbf{x}}_A - \cdots\right), \tag{29}$$

in which we see that each flavor has the expected kinetic term. It is worth recalling that due to the color symmetry that we expect, the coordinate to which the symmetry transformation should apply is $\tilde{\mathbf{X}}$.

In [11] a conceptual relation between the use of the matrix coordinates for non-Abelian gauge theory purposes and the ideas concerned in special relativity is mentioned; see also [5,4,2]. According to an interpretation of the special relativity, it is meaningful if the "coordinates" and the "fields" in a theory have some kind of similar characters. As an example, we observe that both the space-time coordinates x^μ and the electromagnetic potentials $A^\mu(x)$ transform equivalently (i.e., as a $(d+1)$-vector) under boost transformations. Also by this interpretation, the superspace formulations of supersymmetric field and superstring theories are the natural continuation of the special relativity program. In the case of the use of matrix coordinates, it may be argued that the relation between "matrix coordinates" and "matrix fields" (gauge fields of a non-Abelian gauge theory) is one of the expectations which is supported by the spirit of special relativity. From the previous discussion we recall that

(1) the matrix character of gauge fields is the result of dependence of them on matrix coordinates [11],

(2) the symmetry transformations of gauge fields are induced by the transformations of the matrix coordinates [11],

(3) the transformations of fields in the theory on the matrix space appeared to be similar to those of non-Abelian gauge theories; see the relations (12) and (17). This interpretation leads us to conclude that the non-Abelian gauge fields in a confined theory do not have an independent character, and they are introduced into the formalism due to the functional dependence on the matrix coordinates of "bounded quarks". It seems very interesting when we note that by the present experimental data, the existence of pure gluonic states, so-called glueballs, is quite doubtful. This lack of detection may be taken as support for the interpretation presented above.

Acknowledgements. The author is grateful to A. Shariati, and specially to M. Khorrami for helpful discussions. The comments on the manuscript by Gh. Exirifard, and specially by S. Parvizi and M.M. Sheikh-Jabbari are acknowledged.

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